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How good is the bond market at forecasting the risk-free rate?

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Finalmente agradecer a mi supervisor, Mikel Esnaola, por ser una constante inspiración y guía, no solo en el mundo académico, sino en todos los ámbitos.

I declare that this thesis has been composed solely by myself and that it has not been submitted, in whole or in part, in any previous application for a degree. Except where it is stated otherwise by reference or acknowledgment, the work presented is entirely my own.

Jorge Alfredo Pellegrini Padilla

ABSTRACT

The aim of this paper is to determine if the implied forward rates derived from the bond market are a good indicator of the future risk-free rate and understand why discrepancies between these two variables may occur. The fundamental principles of finance will be reviewed in order to explain how time and risk affect the value of money. Then, it will be shown why government' securities are often used as risk-free assets, and therefore, their rate of return is used as the risk-free rate of return in most of the valuation methods. Finally, a quantitative analysis will be performed using data from the U.S. Board of Governors of the Federal Reserve System and Survey of Professional Forecasters. It will find that the accuracy of implied forward rates depends on the length of the forecast and the type of bond. Therefore, taking that into account combined with the fact that implied forward rates data can be easily accessed, this paper concluded that implied forward rates can be used as a reliable indicator of the future risk-free rate if time and resources are a constraint.

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1. INTRODUCTION

Finance studies the management of money, that is, how to save, spend or invest monetary resources. Depending on the type of economic agent, it is possible to differentiate:

- **Personal Finance**: For individuals and families.
- **Corporate Finance**: For companies.
- **Public Finance**: For public entities, such as governments.

Money, also called capital in the corporate world, is exchanged in financial markets. In them, it is possible to buy and sell different financial assets whose prices are defined by the law of supply and demand. The objective of the financial market is to connect agents which need financial resources (borrowers) with those which have financial resources available (lenders) and are willing to lend these resources in exchange for a reward.

The initial motivation of this paper was conceived around June 2019, before I graduated in economics. At that time, I obtained my first job as a financial advisor. The aim of my job was and still is to help and guide my clients, most of them companies, make the best financial decisions. Therefore, buying and selling financial assets is a constant preoccupation of mine.

The most important thing when buying or selling financial assets is to know their value (not to be confused with price). Without a value, it is not possible to know if the client is getting a good or a bad deal. In order to determine the value and eventually set a price, valuation models are used. Unfortunately, there is not a single valuation model that can be used for all assets. Different assets have different characteristics and situations, and thus, they cannot be valued in the same way.

However, valuation models share a common characteristic that called my attention since the first moment I worked with them: their starting point is a theoretical risk-free rate. Once the risk-free rate is defined, one or more risk premiums are added in order to risk-adjust the value of the asset. Hence, the risk-free rate is used as a benchmark for the rest of interest rates, and a small change in it leads to a huge change in the output: the value of the asset.

Being aware of the importance of the risk-free rate in any valuation model, I decided to research further into the possible ways to determine and forecast the value of the risk-free rate. I found out that the most popular way to do it is to observe the bond market and its implied forward rates. The aim of this paper is to determine how accurate the implied forward rates are at forecasting the future risk-free rate and therefore, conclude if they should be used in valuation models over other methods of prediction.

First, I will outline the fundamental principles of finance and the basics of the bond market. I will explain key technicisms of the bond market, how bonds work, why they are used for determining the risk-free rate and then the mathematical relationship between spot and forward rates.

Then, I will focus on the American bond market. I used data from the U.S. Board of Governors of the Federal Reserve System to compute the implied forward rates of zero-coupon bonds with different maturities using the mathematical relationship between spot rates and forward rates. Then, I will compare these implied forward rates with the actual future spot rate to compute the possible forecasting errors that may arise. Once I have the errors, I perform a statistical analysis applying a linear Mixed Effect model to get conclusive results.

Finally, I will try to explain why those errors happen performing a multivariate linear regression model using the data from the Survey of Professional Forecasters conducted by the Federal Reserve Bank of Philadelphia and data from the U.S. Board of Governors of the Federal Reserve System as well.

2. FINANCE: THE BASICS

As I previously mentioned, in short, finance studies how different economic agents save, spend and invest their money. In this paper, I will focus mostly on the business view of finance, that is, Corporate Finance.

From a financial point of view, there are two fundamental challenges that any business decision involves (Lo, 2008):

1) **Asset Valuation**: As its name suggests, it consists of establishing the value of the assets which is affected by the decision. This is the most challenging part of any financial decision.

2) **Asset Management**: Once the value is established; the management is relatively easy to undertake. The option that maximizes the benefits to a business according to its objectives will be chosen.

The core of this paper focuses on the first step: asset valuation. But before I get to the main subject, I will quickly summarize the fundamental principles of finance which are essential in understanding how finance works.

2.1 Fundamental principles of finance

Although there is no consensus on how many principles are fundamental, four are always mentioned since they are the basic building blocks of the financial theory as it is known nowadays. These principles are (Irons, 2019):

1) Behavior of individuals in the market.
2) Cash flows are the main source of the asset's value.
3) The time value of money.
4) The risk-return relationship.

I will now explain each principle in detail.

2.1.1 Behavior of individuals in the market

The principle says that, *ceteris paribus*, individuals' choices satisfy three characteristics:

- **Non-Satiation**: Individuals prefer more money than less money.
- **Risk-Aversion**: Individuals prefer less risk than more risk.
- **Impatience**: Individuals prefer money now than money tomorrow.

Financial theory considers these three characteristics universal to all the agents that operate in the financial markets.

2.1.2 Cash flows are the source of value

The actual movements of cash within a company are represented by cash inflows, which involve an increase of the available cash, and the cash outflows, which involve a decrease of the available cash. The difference between cash inflows and cash outflows is the cash flow.

In fact, cash flows are so important for companies that they are included in one of the four well-known mandatory financial statements: *The Cash Flow Statement*. It is important not to confuse revenues or expenses (reflected on the *Income Statement*) with a cash inflow or cash outflow (reflected on the *Cash Flow Statement*). Let me clearly distinguish the two with an example situation. Let us say:

- The *Cash and Cash Equivalents* (CCE) of the company are initially $0.
- The company sells $1,000,000 of iron, but the payment will be received in one year (It is a revenue, but not an immediate cash flow).
- The cost of the iron is $400,000, which requires an immediate payment to the supplier. (It is an expense and an immediate cash flow).

In the *Income Statement* the company will have a net profit of $600,000, so initially investors may think the company is in a financially healthy position. However, the *Cash Flow Statement* paints a different picture: No cash has been received nor will be received for one year, but the company has an obligation to pay $400,000 immediately. Therefore, the company may be facing bankruptcy due to lack of liquidity.

Cash flows are often used in corporate finance for three purposes (Hales & Orpurt, 2012):

1) **Determine the value of a project/asset**: Generally, *ceteris paribus*, the project/asset that generates more cash flow will be the most valuable.
2) **Determine the liquidity of a business**: As it has been explained in the example above, liquidity is key for any business. Liquidity is determined by cash inflows and outflows, and not by sales and expenses.
3) **Determine the risk of financial products**: Cash requirements, default risk, re-investment requirements, and others are important when determining risk.

Since this paper is focused on the valuation of financial assets, I will only focus on the first purpose, determining the value of a project/asset.

2.1.3 The time value of money

The value of money is not constant. In simple terms, a dollar today is worth more than a dollar tomorrow.

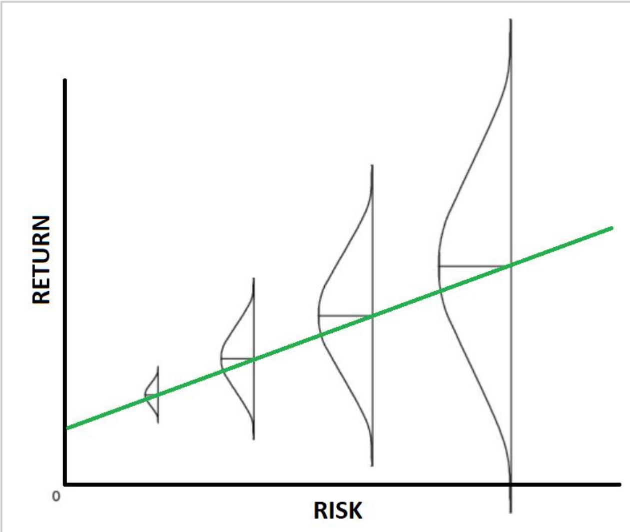
This fundamental principle has tremendous implications. This is the *raison d’etre* of financial markets. Since money today is more valuable than money tomorrow, borrowers are willing to pay a premium, the interest rate, for having the right to spend money today. And conversely, lenders are willing to give up such a right in exchange for compensation, that is, the interest rate.

2.1.4 The risk-return relationship

In general, individuals are risk averse, hence they prefer to avoid risk. Therefore, in order to encourage individuals to take risks, they need to receive a reward.

Although normally this relationship is graphically represented just by a straight upward line representing the positive relationship that exists between risk and return, I prefer the representation that *Howard Marks* gives us in his book *The Most Important Thing* (Marks, 2013)*:*

Figure 1. The risk-return relationship



Source: The Most Important Thing Illuminated, by Howards Marks.
Link: https://www.jstor.org/stable/10.7312/mark16284

In Figure 1 the positive relationship between risk and return can be observed. The higher risk a financial decision has, the higher return it has to offer to the investor. Furthermore, the underlying reason to demand a greater return from the investor's point of view can be also observed: the higher risk it takes, the greater the uncertainty of the return, the possibilities become wider, and the possible loss becomes higher, as well as the benefit.

2.2 The two dimensions of finance: Time and Risk

As simple as it sounds, most of finance's complexity can be reduced to two factors: time and risk.

It is important to bear in mind that past events and outcomes may be really insightful when making decisions, that is why historical data is nowadays so important. Unfortunately, although the past can be known with fair certainty, decisions cannot be taken to change it.

Therefore, financial decisions always involve dealing with the future, and only with the future. As opposed to the past, decisions can be taken now to alter future events, but it cannot be known with certainty. Here is where time and risk come to the fore.

2.2.1 Time

Assets' valuations always deal with the future; hence time is a key variable. As I have already stated, time has direct effects on money: the longer it is needed to receive the money, the lower its value. That is the fundamental principle of Time Value of Money.

The other factor that affects the value of money is risk. Therefore, if one of these factors could be isolated, it would be relatively easy to compute the effects the other factor has on the value of money. Fortunately, it is possible to isolate risk and its effects. In order to do so a risk-free asset which makes payments forward in time is needed.

Once the risk-free asset has been selected, by observing its price and future cash flows, it is possible to formulate a risk-free rate to discount its cash flows. Such rate will only reflect the effects of time on the money's value, leaving aside the effects of risk.

Since this is the main topic of the paper, I will discuss in more depth what a risk-free asset is later.

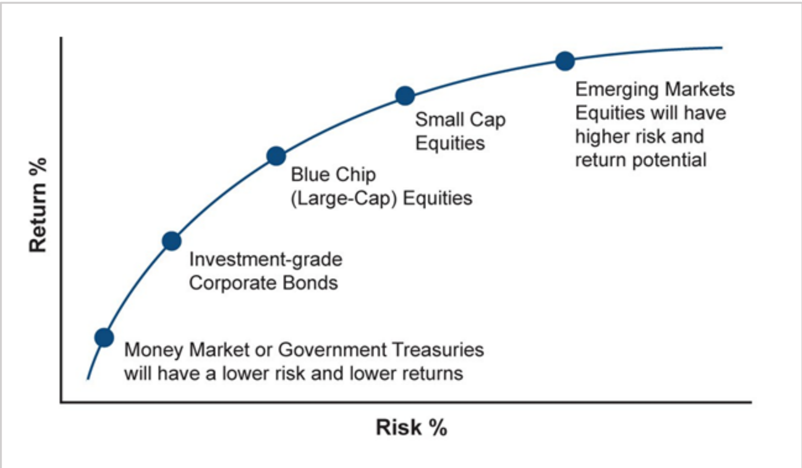
2.2.2 Risk

Risk could be described as the level of uncertainty of the cash flows that an asset may produce. Here it is important to differentiate between expected cash flows and actual cash flows. If cash flows are certain then expected and actual cash flows will coincide and no risk will be implied. But generally, since most financial assets have some degree of risk, expected cash flows differ from actual cash flows.

For example, let us suppose an investor bought American Airlines' shares at the beginning of 2020 expecting to receive dividends at the end of that year. However, due to COVID-19, these expected dividends will now not materialize anymore. In such a case, the expected cash flows do not coincide with the actual cash flows.

Financial markets offer a wide range of financial assets, each one with its own risk level. Although this paper is not focused on risk management, it will be beneficial for the reader to have a grasp on what the most popular assets are classified by their risk level. That is exactly what Figure 2 represents:

Figure 2. Most popular financial assets classified by their risk



Source:https://www.motilaloswal.com/article.aspx/1844/Understand-your-risk-return-trade-off-before-investing-in-equities..

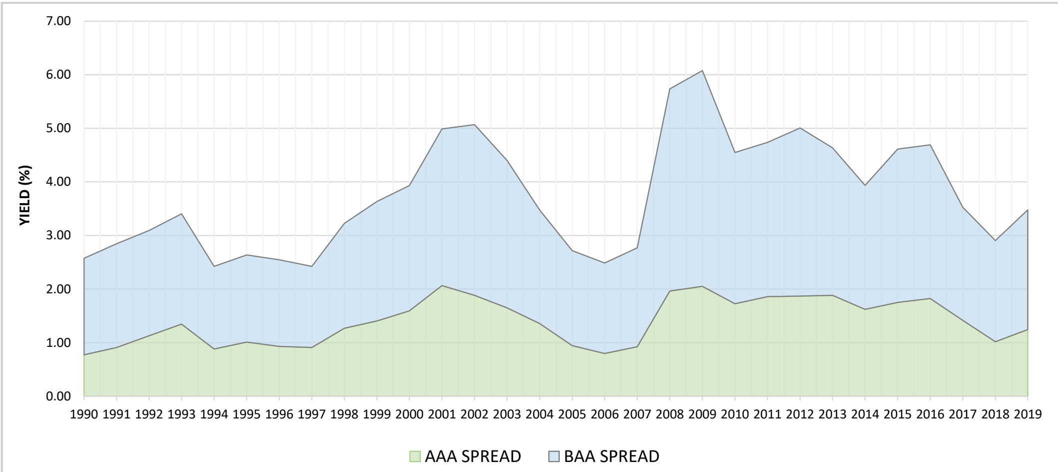
As mentioned at the beginning of the introduction, most valuation models use a risk-free asset as a benchmark. Riskier investments' returns are measured against the risk-free rate, which is nothing more than the yield of the risk-free asset, adding one or more risk premiums depending on the valuation model.

For example, let us take a look at an example observing the following data:

- Risk-free rate: The 10-year treasury bond yield of the US.
- Low-risk corporate bond: Moody's AAA corporate bond yield.
- Mid-risk corporate bond: Moody's Baa corporate bond yield.

The difference between the risk-free rate and corporate bond yields is called the yield spread. This spread is the risk-premium that investors demand to corporate bonds since these companies are more likely to default on their payments than the government.

Figure 3. Yield Spread



Source: Federal Reserve Economic Data. Link: https://fred.stlouisfed.org/graph/?g=r3nM

In Figure 3 the yield spread of the Moody's AAA and BAA corporate bonds against the 10-year treasury bond yield of the US can be observed. Consistently with the risk-aversion assumption, the yield spread is higher for riskier corporates (BAA) than for safer corporates (AAA). Hence, investors demand more return in exchange for facing extra risk. It should be stressed that the yield spread is expressed relative to the risk-free rate.

Finally, it is worth mentioning two key points regarding risk:

1) **The importance of measuring risk properly**: investors demand more return to riskier assets and therefore their valuation is based on the risk-level of the implicit asset. If the risk-level is incorrectly measured, investors will be paying more or demanding less return than they should.
   On occasions, the whole financial system can stagger when a systematic risk-measurement error takes place. A good example of this is the subprime mortgage meltdown of 2008. The root of the problem was that many *Mortgage-Backed Securities* (MBS) were rated as low-risk securities when they actually were much riskier. When all those MBS's held by big investment banks were suddenly down-

rated, they lost most of their value, causing incalculable losses and sowing distrust among the financial system.

2) **Difficulties of measuring risk**: Although the importance of measuring risk properly is well-known, it does not mean that it is an easy task. In fact, modelling the future taking into account all possible outcomes and including the unpredictable human behavior seems almost impossible.

   Fortunately, nowadays technology together with a strong knowledge of statistics and mathematics seem to deliver more and more accurate predictions. Forecasting the future may not be possible, but if forecasts are correct more than 50% of the time, it will definitely make a difference.

3. BOND MARKET: THE RISK-FREE RATE

All asset valuation models have a strong common assumption: investors always have the possibility to invest in a risk-free asset. The return of such an asset is then used as the risk-free rate of return. This allows to get rid of the risk-dimension of the assets, allowing the models to isolate the time-dimension of the valuation. Therefore, the risk-free rate only represents the time value of money. In other words, the risk-free rate is the return that investors receive from simply waiting to receive back their money (Damodaran, 2002).

Then, the risk of the investment is measured relative to the risk-free rate and depending on the model, one or more risk premiums are added to the risk-free rate. This allows the risk-adjustment of the demanded return on a specific investment.

In this section I will explain the requirements for an asset to be considered risk-free, the use of government bonds as proxy for risk-free assets, their characteristics and the relationship between the spot rates and implied forward rates.

3.1 When can an asset be considered risk-free?

In short, an asset can be considered risk-free if its expected returns are equal to its actual returns. That is, if its expected returns are known with certainty. Such a situation can only take place under two conditions (Damodaran, 2002):

- **No default risk**: As its name suggests, it refers to the risk of borrowers being unable to meet their financial obligations. Regardless of how large or safe a company is, default risk is always present to some degree. In general, government securities have no default risk. Taking into account that government securities are loans that private entities lend to the government, in case that the government cannot face the payment at the due date, it can always either print more money or increase taxes to obtain more funds.

  As mentioned, this occurs generally, though there are some governments that have decided to not honor their debts at some point in history. That was the case of Russia in 1998 during the well-known *Russian Crisis*, or the case of Argentina that has already defaulted on its debts eight times in the last twenty years (Mohan, 2016). Usually such situations take place when debt is not denominated in the country's domestic currency. In such scenarios, printing more money or collecting taxes cannot solve the situation, and often a debt-restructuring process has to take place.

- **No reinvestment risk**: An asset faces reinvestment risk when the investor will not be able to reinvest the cash flows that the asset may generate at the same current rate of return. For example, consider a three-year government bond with an annual coupon payment. Although the payments are known with certainty in time, the reinvestment rate at which the yearly coupons can be invested in the future is not known. Therefore, the only fixed-income securities that have no reinvestment risk are zero coupon bonds. (Fernando, 2019). A further example of this is given in the following section.

3.2 Government Bonds, Zero-Coupon Bonds and the Risk-Free Rate

Government Bonds are nothing but loans where lenders are private entities (individuals or private companies) and the borrower is the national government. Hence, bond's holders or lenders, are entitled to receive fixed amounts of cash flows at fixed dates for a certain period of time.

The following concepts are essential to understand how government bonds work (Fabozzi, 2009):

- **Bond's Price**: The price that is actually paid to acquire the bond.
- **Principal or Face Value (FV)**: The nominal value stated by its issuer, in this case, the government.
- **Maturity date**: Date at which the principal of the bond is paid.
- **Interest Rate**: The interest rate that is applied to the principal in order to compute the coupons.
- **Coupons**: The interest payments that the bond holder will receive from the issuance of the bond until the maturity date. Coupons, if paid annually, are computed as the product of the annual interest rate and the face value.
- **Yield to Maturity (YTM)**: It represents the return investors receive if the bond is held until maturity. Technically, it is the Internal Rate of Return (IRR) required for the sum of the coupons and the face value to equal the bond's price.

Since those technicisms are key to understand this paper, let us see a numerical example: Suppose that Spain is auctioning three-year government bonds at $975 with a face value of $1,000 on the 12/31/2020. The annual interest rate is 4.00% and the coupons are paid annually. Therefore:

- Bond's Price = $975
- Face Value = $1,000
- Interest Rate = 4.00% (annual)
- Coupons = 4.00% * $1,000 = $40 (paid annually)
- Maturity date = 12/31/2023 (three years)

With this data, it is possible to calculate the Yield to Maturity:

$$PRICE = \frac{Coupon}{(1 + YTM)} + \frac{Coupon}{(1 + YTM)^2} + \frac{Coupon + Face\ Value}{(1 + YTM)^3}$$

$$\$975 = \frac{\$40}{(1 + YTM)} + \frac{\$40}{(1 + YTM)^2} + \frac{\$40 + \$1{,}000}{(1 + YTM)^3}$$

$$YTM = 4.917\%$$

Is the YTM in this case a good proxy for the three-years risk-free rate? Technically, it is not. As commented previously, this three-year bond has no default risk but it does have reinvestment risk: it is not known, and therefore uncertain, at which rate the coupons payments could be reinvested in the future.

The only type of government bonds that has no reinvestment risk is the zero-coupon bond. These are bonds which have no coupon payments. In fact, they only generate one payment: their face value at the maturity date. The YTM of zero-coupon bonds are the best estimate for the risk-free rate since they are the most similar to a risk-free asset in practical terms.

However, few pure zero-coupon bonds exist in the American bond market. Roughly, the Primary American bond market is composed of:

- **Treasury Bills** (T-bills): Short-term debt bonds with maturities of a year or less without interest payments (no coupons). Treasury Bills are issued in maturities of 4, 13, 26 and 52 weeks.
- **Treasury Notes** (T-Notes): Medium-term debt bonds with maturities between a year and ten years and biannual interest payments (coupons). Treasury Notes are issued in maturities of 2, 3, 5, 7 and 10 years.
- **Treasury Bonds** (T-Bonds) Long-term debt bonds with maturities between ten and thirty years and biannual interest payments (coupons). Treasury Bonds are issued in maturities of 20 and 30 years.

As it can be observed, the only securities that can be considered zero-coupon bonds are Treasury Bills. Such securities have a maximum maturity of one year. Therefore, investors who need to compute the risk-free rate for longer periods do not have the proper risk-free asset to do so.

Fortunately, the U.S. Treasury introduced the Separate Trading of Registered Interest and Principal of Securities (STRIPS) back in 1985. In short, STRIPS are created detaching the interest payments from bonds, creating as many STRIPS as coupon payments and one for the face value payment. The original bond has to have a minimum maturity of 10 years in order to be eligible to transform it into a STRIP. For example, a 10-year bond with annual payments can be *stripped* down to eleven STRIPS: ten annual coupon payments and one including the face value payment. Therefore, STRIPS are zero-coupon bonds, and their yield to maturity can be used as a proxy for the risk-free rate (Sack, 2000).

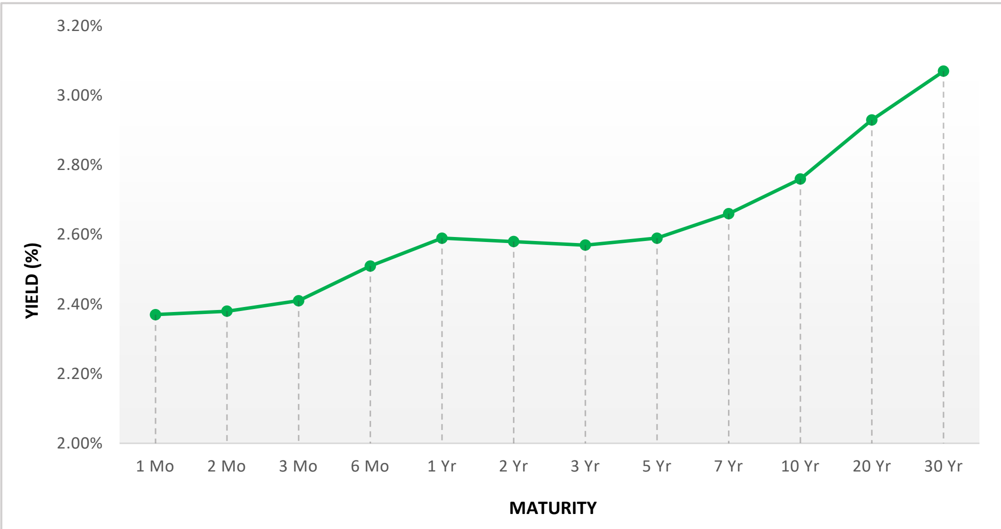
Furthermore, zero-coupon yields can also be estimated from Treasury Notes and Bonds using statistical models. The most widely used methods are the Nelson-Siegel method (Nelson & Siegel, 1987) and the Fisher-Nychka-Zervos method (Fisher, Nychka, & Zervos, 2000). The methodologies and characteristics of both methods are not the concern of this paper, but it is worth highlighting that contemporary variations of both models exist, that aim to improve some of their weak points, such as the Nelson-Siegel-Svensson method (1994).

3.3 The Yield Curve

The so-called Yield Curve or Term Structure of Interest Rates can be constructed by plotting yield in the Y-axis and different bond maturities in the X-axis. If zero-coupon yields are used, then it is called the zero-coupon yield curve.

The U.S. Department of the Treasury publishes the Yield Curve of the outstanding bonds daily on their webpage.

Figure 4. U.S. Yield Curve at May 23, 2019



Source: U.S. Department of the Treasury.
Link: https://www.treasury.gov/resource-center/data-chart-center/interest-rates/

The Yield Curve provides a snapshot of the market's expectation of the future spot rate and economic landscape through the relationship of short and long-term yield. Depending on the slope of the line, it is possible to differentiate:

- **Normal Yield Curve** (Upward sloping curve): The positive slope shows that lending for longer periods of time should be rewarded more than lending for shorter periods of time. Longer periods of time mean longer exposure to unexpected events, and therefore higher risk. This is how a rational market should behave.
- **Inverted Yield Curve** (Downward sloping curve): When long-term interests fall behind short-term interests the Yield Curve may be perceived as an indicator of a future economic downturn.
- **Flat Yield Curve**: This type of curve is the transition between a normal Yield Curve and an Inverted Yield Curve, confirming that an economic slowdown may take place soon.

- **Steep Yield Curve**: Historically when long-term yields rise much faster than short-term yields, it means that an economic period of expansion is about to take place.
- **Humble Yield Curve**: This shape indicates that the middle-term yield is higher than short- and long-term yields. This typically indicates a slowdown of economic growth.

At this point it is important to remember that a Zero-Coupon Yield Curve would represent the market's expectation of the risk-free rate, which as I have already stated several times, is the basic building block for valuations and riskier interest rates. In the following section I will explain how to mathematically compute these market's expectations to eventually check, through a quantitative analysis and historical data, if such expectations/forecasts are an accurate indicator of the future risk-free rate.

3.4 The relationship between Spot Rates and Forward Rates

3.4.1 Spot Rate

The spot rate can be defined as the theoretical rate of return that investors can achieve at the current moment. The spot rate for a zero-coupon bond equals its Yield to Maturity or its Internal Rate of Return. It can be expressed as follows:

$$P = \frac{FV}{(1+s)^n}$$

$$(1+s)^n = \frac{FV}{P}$$

$$1+s = \left(\frac{FV}{P}\right)^{\frac{1}{n}}$$

$$s = \left(\frac{FV}{P}\right)^{\frac{1}{n}} - 1$$

Where:

- $s =$ Spot Rate
- $n =$ Years until Maturity
- $FV =$ Face value
- $P =$ Current Zero-Coupon Bond Price

3.4.2 Forward Rate and Future Spot Rate

Keeping the definition of spot rate in mind, the forward rate is a current forecast about rate of return that is expected to be applied at some point in the future. The most common notation for forward rates is as follows:

$$Forward\ Rate\ (j,k) = {}_jf_k$$

Where:

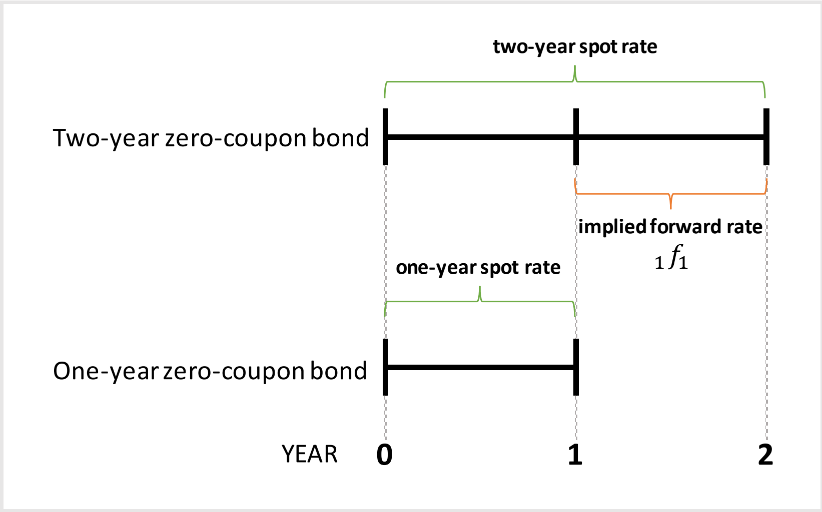
- $j = length\ of\ the\ forward\ period\ from\ today$
- $k = time - to - maturity\ of\ the\ bond$

On the other hand, the future spot rate is the actual rate that will be applied at some particular time in the future with certainty.

3.5 The Law of One Price and Implied Forward Rates

The Law of One Price states that two identical assets which deliver the same identical cash flows at the exact same time, have to be priced identically in well-functioning markets. Therefore, their cash flows have to be discounted at the same rate of return. If such condition does not hold, arbitrage opportunities may arise. (Brealey, Myers , & Allen, 2016)

Figure 5. Implied Forward Rate



Source: Own elaboration

As Figure 5 represents, the Law of One Price suggests that it is possible to derive a forward rate from two different spot rates, that is, the implied forward rate.

Under the different ways of calculating the interest rate, the formula of the implied forward rate can take different forms.

3.5.1 Implied Forward Rate: Simple rate

Under simple interest rate and assuming that the Law of One Price holds, the two following strategies have to lead to the same output:

- **STRATEGY 1**: At year 0, invest $1 in a two-year zero-coupon bond. At year 2, the bond will mature and investors will receive their initial investment in addition to the interests using the two-year spot rate.

- **STRATEGY 2:** At year 0, invest $1 in a one-year zero-coupon bond. At year 1, the bond will mature and investors will receive their initial investment plus the interests using the one-year spot rate. Immediately, at year 1, investors will reinvest what they just received (initial investment + interests) in another one-year zero-coupon bond. At year 2, the bond will mature and investors will receive their initial investment along with the interests using the one-year implied forward rate ($_1f_1$).

Such equality can be formally expressed as follows:

$$\underbrace{1 + s_2 * n_2}_{\text{STRATEGY 1}} = \underbrace{(1 + s_1 * n_1) * \left(1 + {}_1f_1 * (n_2 - n_1)\right)}_{\text{STRATEGY 2}}$$

Where:

- s_1 = one-year spot rate
- s_2 = two-year spot rate
- n_1 = time period expressed in years for the one-year spot
- n_2 = time period expressed in years for the two-year spot
- ${}_1f_1$ = one year forward rate, 1 year from today

If the implied forward rate is isolated, the following equation is obtained:

$$ {}_1f_1 = \frac{1}{n_2 - n_1} * \left(\frac{1 + s_2 * n_2}{1 + s_1 * n_1} - 1\right)$$

3.5.2 Implied Forward Rate: Yearly compounded rate

Following the previous example under the yearly compounded interest rate method, such equality can be expressed as follows:

$$\underbrace{(1 + s_2)^{n_2}}_{\text{STRATEGY 1}} = \underbrace{(1 + s_1)^{n_1} * (1 + {}_1f_1)^{n_2 - n_1}}_{\text{STRATEGY 2}}$$

If the implied forward rate is isolated, the following equation is obtained:

$$ {}_1f_1 = \left(\frac{(1 + s_2)^{n_2}}{(1 + s_1)^{n_1}}\right)^{\frac{1}{n_2 - n_1}} - 1$$

3.5.3 Implied Forward Rate: Continuously compounded rate

Following the previous example under the yearly compounded interest rate method, such equality can be expressed as follows:

$$\underbrace{e^{(s_2 * n_2)}}_{\text{STRATEGY 1}} = \underbrace{e^{(s_1 * n_1)} * e^{\left({}_1f_1 * (n_2 - n_1)\right)}}_{\text{STRATEGY 2}}$$

If the implied forward rate is isolated, the following equation is obtained:

$$ {}_1f_1 = \frac{(s_2 * n_2) - (s_1 * n_1)}{n_2 - n_1}$$

4. QUANTITATIVE ANALYSIS

4.1 Hypothesis

As it has been reviewed through this paper, the value of any asset lies in its cash flows. Time and risk implied on these cash flows determine their value, and therefore, determine the value of the asset. Most valuation methods first isolate the effects of time using a risk-free rate of return. Then, one or more risk-premiums are added in order to capture and adjust the value of the cash flows to the degree of risk that the asset has.

The risk-free rate of return is a theoretical rate at which investors can invest their money with zero-risk level. In practice, there is no investment, entity or financial asset that has no risk. Within financial markets, zero-coupon government bonds are the most similar to the definition of a risk-free asset, and therefore, their rate of return is used as the risk-free rate.

By observing current zero-coupon spot rates investors can derive the implied forward rates from them. These are forecasts about the future risk-free rate made by all the participants of financial markets.

The aim of this quantitative analysis is to check if the implied forward rates are accurate predictors of the future spot rates. Furthermore, it will also try to explain why forecasting errors may occur.

4.2 Case Study and Data

The quantitative analysis will focus on the U.S. bond market. Fortunately for my analysis, the Federal Reserve System is well-aware of the key role that the risk-free rate has in finance and therefore supplies plenty of free historical data to any user that is interested in it. Moreover, their division of Research & Statistics and Monetary Affairs computes the zero-coupon yield curve on a daily basis. (Gürkaynak, Sack, & Wright, 2007)

The valuable data that such source provides has the following characteristics:

- Daily estimates of the off-the-run zero-coupon yield curve based on outstanding treasury notes and bonds.
- The Nelson-Siegel-Svensson method is used to compute the estimates.
- It is based on continuous compounding convention.
- Maturity range of the zero-coupon bonds: from 1 year to 30 years.
- Historical data of the zero-coupon bond yield curve from 1961 to 2020:
    - Yield of one to seven years zero-coupon maturity bonds from June 1st,1961.
    - Extended to ten years zero-coupon maturity bonds from August 16th, 1971
    - Extended to fifteen years zero-coupon maturity bonds from November 15th, 1971.
    - Extended to twenty years zero-coupon maturity bonds from 2 July 1981
    - Extended to thirty years zero-coupon maturity bonds from November 25th 1985.
- For more information on the data: https://www.federalreserve.gov/pubs/feds/2006/200628/200628pap.pdf

- The data can be downloaded with the following link: https://www.federalreserve.gov/data/yield-curve-tables/feds200628_1.html

On the other hand, the second part of the quantitative analysis will try to explain why differences between implied forward rates and future spot rates may happen. I used data from the Survey of Professional Forecasters conducted by the Federal Reserve Bank of Philadelphia. This survey provides historical data on the forecast of the most important macro-economic indicators from 1968 until today. The characteristics of the data I used for my analysis are:

- Historical values and one-year forecasted values of the following indicators:
  - Consumer Price Index (CPI)
  - Nominal GDP (NGDP)
- Since the data is released every quarter, I am using the arithmetic mean of the four quarters in order to have a single value expressed on an annual basis.
- For more information on the data: https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/historical-data/mean-forecasts

Furthermore, I also used data from the U.S. Board of Governors of the Federal Reserve System in order to obtain historical data about the Federal Funds Rate:

- Historical values of the following variables:
  - Effective Federal Funds Rate (FEDFUNDS)
  - Federal Funds Target Rate (until 2008) (DFEDTAR)
  - Federal Funds Target Range – Upper Limit (DFEDTARU)
  - Federal Funds Target Range – Lower Limit (DFEDTARL)
- Expressed on annual basis
- For more information on the data: https://fred.stlouisfed.org/graph/?g=rbSC

4.3 Methodology

In this section I will explain the methodology I used to perform my analysis. Broadly, the analysis can be divided into three steps:

1) Determine the implied forward rates and compare them with the actual future spot rates to observe the errors that may arise.

2) Define the linear mixed effect model in order to perform a statistical analysis.

3) Explain which variables may affect the implied forward rates' errors using a multivariate linear regression model.

4.3.1 Implied Forward Rates

The historical data set provided by the Federal Reserve System contains the spot rates of 30 different types of zero-coupon bonds: ranging from 1 year to 30 years maturity. The nomenclature used to differentiate them is as following:

- SVENY01: One-year maturity zero-coupon bond
- SVENY02: Two-year maturity zero-coupon bond
- SVENY03: Three-year maturity zero-coupon bond

- …
- SVENY30: Thirty-years maturity zero-coupon bond

The part of the nomenclature “SVEN” indicates that the Nelson-Siegel-Svensson method was used to estimate the zero-coupon spot rates, and the “Y##” indicates the maturity years. In addition, although the data is provided on a daily basis, it is also possible to download the data set on a yearly basis. I decided to select the latter for simplicity purposes.

The implied forward rates for the zero-coupon bonds ranging from 1 to 5 years have been calculated using the full range of zero-coupon bonds. That means that when the data has no missing values (from 1985 onwards), in a certain year it is possible to calculate the following number of implied forward rates for each type of bond:

- 29 Implied forward rates for SVENY01
- 14 Implied forward rates for SVENY02
- 9 Implied forward rates for SVENY03
- 6 Implied forward rates for SVENY04
- 5 Implied forward rates for SVENY05

Taking this into account and that at the beginning of the sample some values were missing, I was able to calculate a total of 2,841 implied forward rates:

- 1,305 for SVENY01
- 630 for SVENY02
- 405 for SVENY03
- 276 for SVENY04
- 225 for SVENY05

Once the implied forward rates were calculated for each year and for each type of bond, they were compared to the actual future spot rates. I calculated the difference between these two values, the implied forward rate and the future spot rate, in absolute values.

Table 1. Implied forward rates for SVENY01 from 1961 to 1970

		PREDICTED YEAR															
		1961	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976
YEAR PREDICTED FROM	1961	3.14	3.70	4.04	4.19	4.20	4.12	3.97									
	1962		3.02	3.33	3.58	3.78	3.94	4.07	4.17								
	1963			3.62	4.01	4.13	4.14	4.14	4.14	4.14							
	1964				3.75	4.04	4.18	4.19	4.19	4.19	4.19						
	1965					4.73	4.77	4.77	4.77	4.77	4.77	4.77					
	1966						4.76	4.71	4.71	4.71	4.71	4.71	4.71				
	1967							5.53	5.64	5.74	5.77	5.77	5.77	5.77			
	1968								5.97	6.10	6.20	6.20	6.20	6.20	6.20		
	1969									8.20	8.03	7.80	7.53	7.24	6.93	6.61	
	1970										5.00	5.98	6.29	6.38	6.41	6.42	6.42

Source: Based on RStudio output

As an example, Table 1 shows how the implied forward rates were calculated from 1961 to 1970 for SVENY01. It is important to remember that at that time, the data was only available for bonds up to seven-years maturity zero-coupon bond (SVENY01 to

SVENY07). Therefore, it is possible to only calculate 6 implied forward rates for each period, instead of 29.

In Table 1 it is possible to observe the actual future spot rate highlighted in the color green, and above it, all the implied forward rates that tried to forecast this years' rate from different points in time. The error of each implied forward rate was calculated by measuring the difference between the implied forward rate (in white) and the actual future spot rate (shadowed green) in absolute values.

4.3.2 Linear Mixed Effect model

Once the errors for each of the 2,841 implied forward rates were calculated, I decided to use a linear mixed effect model to perform an econometric analysis. The reason behind such decision is the nature of my data. Linear regression models are often used when the data is coming from a single homogeneous group. However, the implied forward rates' errors have a hierarchical structure.

Therefore, using a linear mixed effect model allowed me to take into account such structure, and eventually difference between fixed and random effects that my dataset may have.

The general expression of a linear mixed effect model in matrix notation is:

$$y = \beta X + uZ + \epsilon$$

Where:

- $y =$ a vector of observations that represents the dependent variable.
- $\beta =$ an unknown vector of fixed effects that needs to be estimated.
- $u =$ an unknown vector of random effects that needs to be estimated.
- $X =$ a matrix of observations that relates β to y.
- Z = a matrix of observations that relates u to y.
- $\epsilon =$ an unknow vector of random errors, also known as the error term.

However, in my analysis, I defined a slightly different model that contemplates a constant random intercept. In matrix notation, the model (mod1.1) can be expressed as follows:

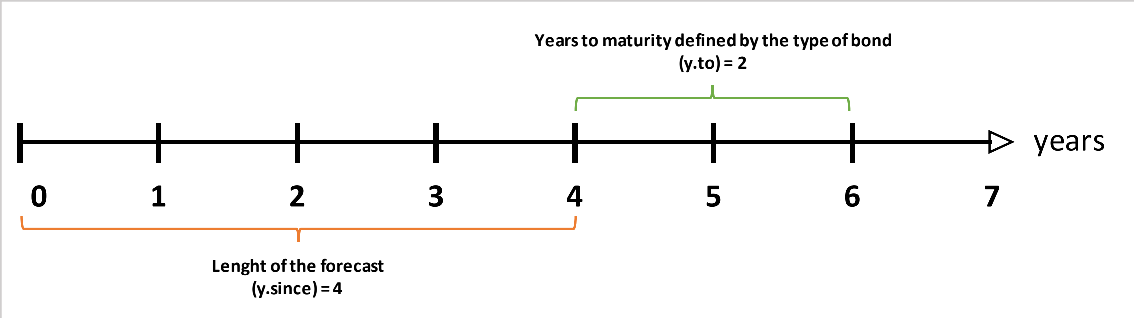
$$(error)_{it} = \beta_1(y.since)_{it} + \beta_2(y.to)_{it} + u_t + \epsilon_{it}$$

Where:

- **(error)**: It is a (2,841 x 1) vector that represents the dependent variable "Implied forward rates' errors" of type of bond (i) at a period (t).
- $\boldsymbol{\beta_1}$ **and** $\boldsymbol{\beta_2}$: They are a (k x 1) unknown vectors of fixed effects that need to be estimated.
- **(y.since)**: It is a (2,841 x k) matrix of observations regarding the length of the forecast, that is, the difference between the forecasted year and the year a forecast is performed.
- **(y.to):** It is a (2,841 x k) matrix of observations regarding the type of bond.

- **u**: It is a (2,841 x 1) vector that represents the random intercept.
- ϵ: It is a (2,841 x 1) vector of random errors.

Figure 6. Future rate of a two-year zero-coupon bond four years forward



Source: Own elaboration

Then I decided to define two more models with slight modifications: the first (mod1.2) to assess the linear relationship between the type of bond (y.to) and the dependent variable (error), and the other (mod1.3) to capture any possible non-linear relationship between the length of the forecast (y.since) and the dependent variable (error) by adding a quadratic term.

Figure 7. Linear Mixed Effect Models code and AIC

```r

## Linear mixed effect models:

require(nlme)


mod1.1 <- lme(fixed = error ~ y.since + y.to,
              random = ~ 1 | year,
              data = ifr, na.action = na.omit)


mod1.2 <- lme(fixed = error ~ y.since + factor(y.to),
                random = ~ 1 | year,
                data = ifr, na.action = na.omit)


mod1.3 <- lme(fixed = error ~ y.since + I(y.since^2) + y.to,
              random = ~ 1 | year,
              data = ifr, na.action = na.omit)

## Akaike information criterion (AIC)

AIC(mod1.1)
AIC(mod1.2)
AIC(mod1.3)

```

Eventually, as it can be observed in Figure 7, I used the Akaike Information Criterion (AIC). Such criterion rewards models for their high goodness-of-fit score and penalize them for their complexity. The lower the value, the better the model.

4.3.3 Possible variables that explain the implied forward rates' errors

Finally, I ran a multivariate linear regression model in order to try to find which variables may explain the errors of the implied forward rates, this is to say, why the forecasts had errors.

In order to perform such analysis, first I downloaded the data of the following variables from the Survey of Professional Forecasters and the U.S. Board of Governors of the Federal Reserve System:

- Consumer Price Index (CPI)
- Nominal GDP (NGDP)
- Federal Funds Rate (FFUNDS)

I selected these three variables since they are the best proxy for the three factors that affect the yield curve the most: inflation, economic growth and interest rate respectively.

Regarding the data from the Survey of Professional Forecasters, it provides the historical values from 1968 until 2020. But what is even more important, the dataset also contains a one year forecast of each variable. Since all the values are given on quarterly basis, I calculated the mean of the four quarters to have a value expressed on an annual basis.

Then using these two values (forecasted and actual) and taking into account the one-year lag, I calculated the error between the forecast and the actual value for the three variables:

- $(\boldsymbol{CPI\_ERROR})_{\boldsymbol{t}} = (F\_CPI)_{t-1} - (CPI)_{t}$
- $(\boldsymbol{NGDP\_ERROR})_{\boldsymbol{t}} = (F\_NGDP)_{t-1} - (NGDP)_{t}$

On the other hand, the U.S. Board of Governors of the Federal Reserve System provides historical data of the Federal Funds Rate. From 1961 to 2008 I computed the difference between the target rate and the effective rate. After 2008, since the FED decided to report the target rate as a range, I computed the average of the upper and lower limit to define a target rate. Then, as I did before, I computed the difference between the target and the effective rate.

Since two out of three variables only have one year forecast one year forward, it was only possible to perform the analysis for the implied forward rates' errors of the one-year zero-coupon bond (SVENY01), one year forward.

Once I had defined all three variables, I performed the statistical analysis. The general expression of a multivariate linear regression model in matrix notation is:

$$y_t = \beta_0 + \sum_{j=1}^{p} \beta_j x_{jt} + \epsilon_t \quad and\ p > 1$$

Where:

- y_t = a vector of observations that represents the dependent variable at period t.
- β_0 = the regression intercept.
- β_{jt} = an unknown vector that needs to be estimated.
- x_{jt} = a matrix of observations that relates β_j to y.
- ϵ_t = an unknow vector of random errors, also known as the error term.

Therefore, once I fit the data on the model, the formal expression in matrix notation of my model is as follows:

$$(error)_t = \beta_0 + \beta_1(CPI\_ERROR)_t + \beta_2(NGDP\_ERROR)_t + \beta_3(FFUNDS\_ERROR)_t + \epsilon_t$$

4.4 Discussion

4.4.1 Findings

Statistical Test 1: Linear Mixed Effect Model

First of all, as I mentioned before, three slightly different models were designed. Then, using the Akaike Information Criterion (AIC) one of them was selected.

Table 2. Akaike Information Criterion: Results

Akaike Information Criterion (AIC)	
Model	**AIC**
Model 1.1	5,998
Model 1.2	6,010
Model 1.3	5,975

Source: Based on RStudio output

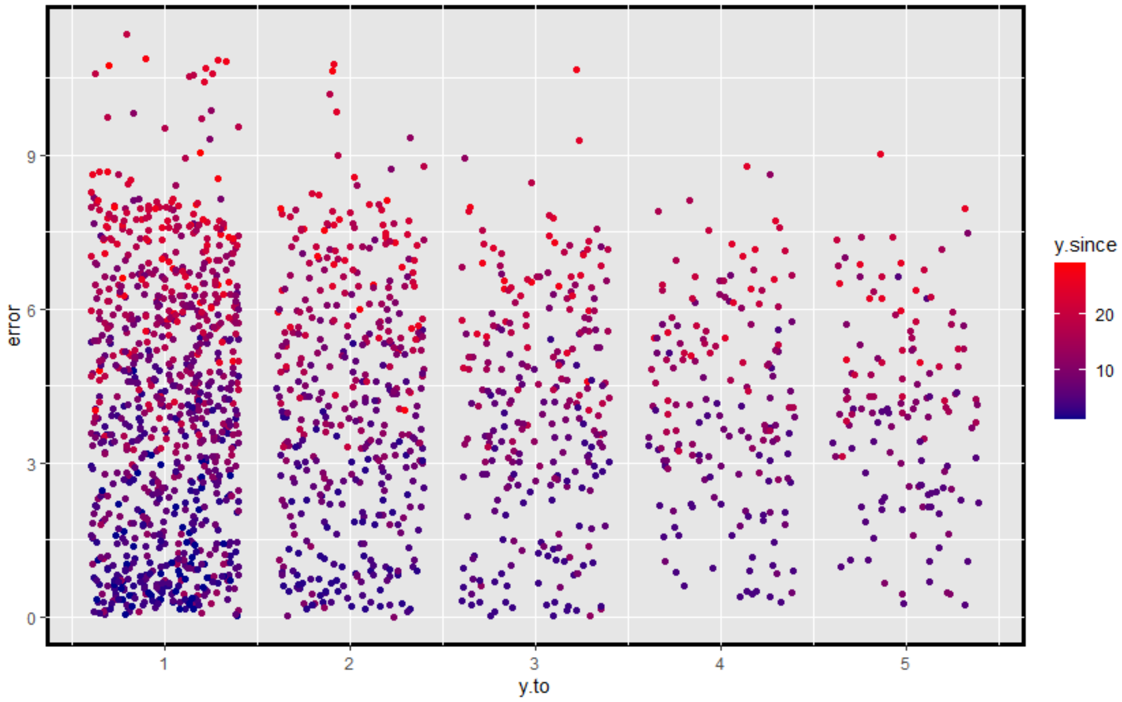
As it can be observed in Table 2, model 1.3 scored the lowest value, meaning that it has a better balance between the ability to fit the data and the complexity of the model than the other. Hence, the results of model 1.2 leads us to believe that a linear relationship between the type of bond (y.to) and the dependent variable (error) may exist. On the other hand, results of model 1.3 suggests that a non-linear relationship between the length of the forecast (y.since) and the dependent variable (error) may exist. Such a non-linear relationship is not captured in model 1.1 but in model 1.3 because a quadratic term was added in the later.

The formal expression for model 1.3 in matrix notation is:

$$(error)_{it} = \beta_1(y.since)_{it} + \beta_2(y.since)^2_{it} + \beta_3(y.to)_{it} + u_t + \epsilon_{it}$$

Once the model was selected, I ran the model in R in order to obtain the statistical results.

Figure 8. Implied Forward Rates' Errors Scatterplot

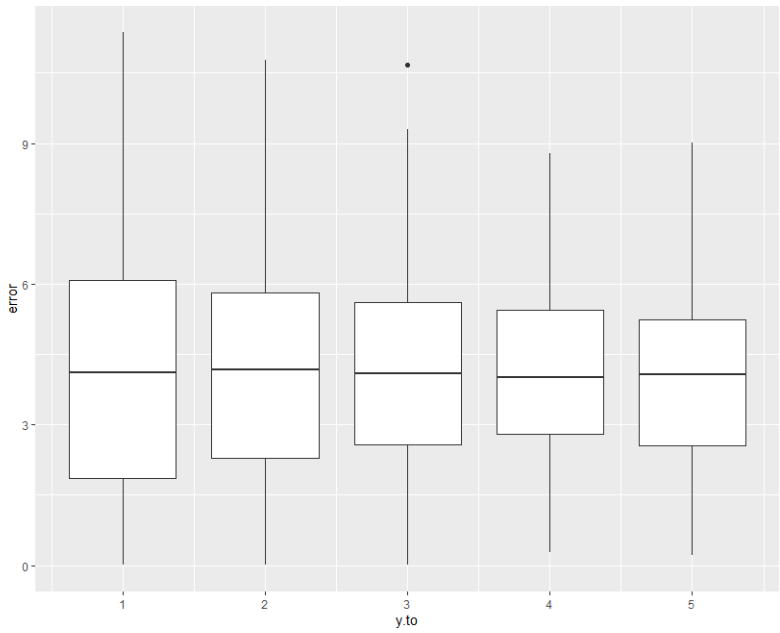


In Figure 8 the implied forward rates' errors are represented in the vertical axis as percentage points. And then the different types of bonds (y.to) are represented in the horizontal axis from one-year maturity to five-year maturity. In addition, depending on the length of the forecast (y.since) the color of the dots vary: the dots are represented by the blue color when the length of the forecast is short, but they turn red as the length lengthens.

It can be observed that most of the blue dots are concentrated in the bottom of Figure 8 while the red dots are concentrated towards the top of the figure. That means that in general errors are lower the shorter the length of the forecast is. Therefore, the one-year implied forward rate for the next year will be more accurate than the one-year implied forward rate 20-years forward.

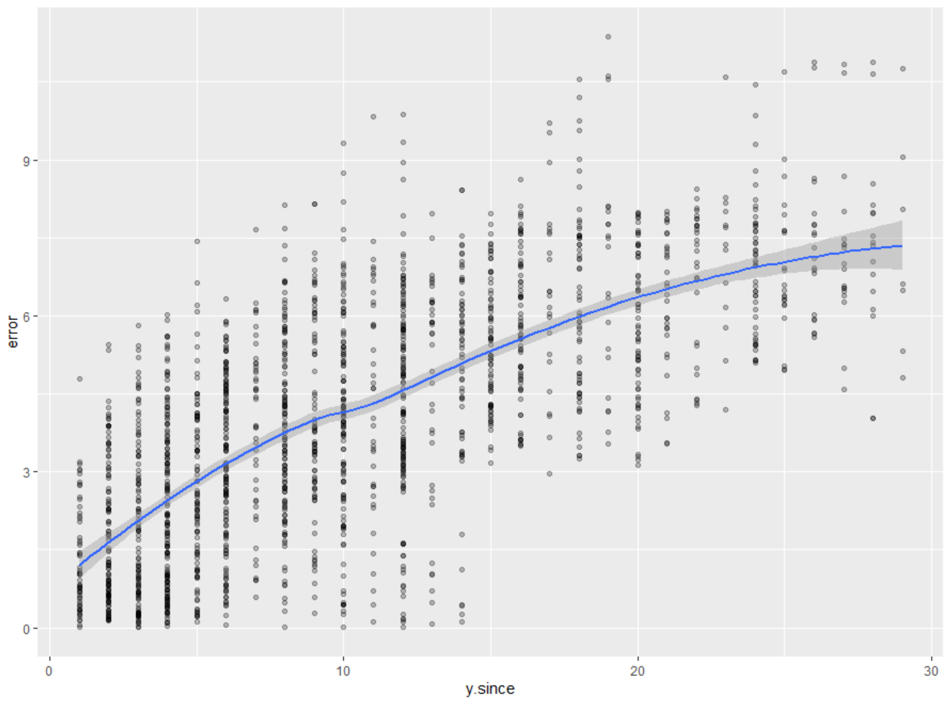
Furthermore, in Figure 8 the negative relationship between the implied forward rates' error (error) and the type of bond (y.to) is represented: the implied forward rates errors' variability is greater the shorter the time to maturity of the bond.

Figure 9. Type of bonds: Boxplot



In order to double check the negative relationship between the implied forward rates error's variability and the type of bond, Figure 9 was designed. Now it can clearly be observed that the lower the time to maturity of the bond is, the greater the variability of errors. This may be a consequence of the bond's ability to smoothen the effects of external shocks. For example, if inflation skyrockets one year, the forecasting errors due to this unexpected shock can be smoothened in a five-year bond forecast meanwhile in a one-year bond it is not possible.

Figure 10. Non-linear relationship



In Figure 10 the relationship between the length of the forecast (y.since) and the dependent variable (errors) is stated. It becomes clear that a positive relationship between these two variables exists: the longer the length of the forecast, the greater the error. Furthermore, the blue curve clearly shows that the relationship is not linear. Due to the addition of a quadratic term on the model, such a relationship can be captured.

Table 3. Linear Mixed Effects Model: estimates

RANDOM EFFECTS	intercept	residual
Std. Deviation	1.2927	1.1303

FIXED EFFECTS	ESTIMATE	LOWER	UPPER	P-VALUE	
intercept	1.2983	0.9293	1.6674	0.0000	***
y.since	0.3225	0.2935	0.3514	0.0000	***
(y.since)^2	-0.0034	-0.0045	-0.0023	0.0000	***
y.to	-0.1331	-0.1738	-0.0925	0.0000	***

Number of observations: 1859

Number of groups: 59

Source: Based on RStudio output

Finally, Table 3 summarizes the parameter estimation's result. First of all, it is important to notice that all the independent variables are significant at all confidence levels. Furthermore, the results mirror the conclusions that were drawn before:

- **Length of the forecast (y.since)**: The positive sign of the estimate describes the positive relationship with the dependent variable: the greater the length of the forecast, the greater the error of the forecast. Moreover, the quadratic term of the variable indicates that the relationship is not linear.

- **Type of bond (y.to)**: The negative sign of the estimate represents a negative relationship with the dependent variable: the more years to maturity, the lower the error of the forecast. Furthermore, the lower AIC's score of model 1.2 indicates that such relationship may be linear.

In conclusion, implied forward rates seem to be more accurate when it comes to long-term bonds rather than short-term bonds. In addition, the accuracy of forecasts is improved when the implied forward rate forecast only a few years in the future rather than longer periods.

Statistical Test 2: Multivariate Linear Regression Model

Table 4. Correlation Matrix

CORRELATION	error	CPI_ERROR	NGDP_ERROR	FFUNDS_ERROR
error	**1.000**			
CPI_ERROR	0.134	**1.000**		
NGDP_ERROR	-0.112	-0.161	**1.000**	
FFUNDS_ERROR	0.662	0.263	0.004	**1.000**

Source: Based on RStudio output

Firstly, I examined the correlation between the variables of the model. A high correlation between the independent variables would indicate multicollinearity issues, which may be fatal for the model. Fortunately, as it can be observed in Table 4, independent variables do not have a strong correlation among them. However, a strong positive correlation between the dependent variable (error) and the independent variable FFUNDS_ERROR exists.

Table 5. Multivariate Linear Regression Model: Results

COEFFICIENTS	ESTIMATE	LOWER	UPPER	P-VALUE	
intercept	0.7304032	0.0606	1.4002	0.0335	*
CPI_ERROR	-0.2002261	-0.6332	0.2328	0.3533	
NGDP_ERROR	-0.0001718	-0.0013	0.0010	0.7667	
FFUNDS_ERROR	0.7345707	0.4431	1.0260	1.35E-05	***

Source: Based on RStudio output

In line with the previous table, Table 5 shows that the only independent variable that is significant at all levels is FFUNDS_ERRORS. Therefore, I can conclude from this analysis that the federal funds rate is a good explanatory variable when it comes to explaining the implied forward rates' error one year forward. In fact, such a conclusion is in line with the Bonds Market behavior: after an increase of the federal funds rate, newly issued government securities also come with an increase in their interest rate. Since the return on these securities is now higher, the risk-free rate increases.

4.4.2 Limitations

The following limitations of the study should be considered:

- Implied forward rates have tremendous relevance in any financial transaction. For instance, they are often used in forward agreements and interest rate swaps. Hence, large entities with practically unlimited funds are investing millions of dollars trying to make accurate forecasts of the future rates with sophisticated methods.

  In my study, due to limitations on time and resources, I used a linear mixed effect model which may be not the most sophisticated nor the most accurate model for panel data. However, similar results were obtained in line with previous academics' results.

- The historical data on the zero-coupon yield curve provided by the U.S. Board of Governors of the Federal Reserve System was used for calculating the implied forward rates. The Federal Reserve System uses the Nelson-Siegel-Svensson method to estimate the zero-coupon yield curve.

  It should be noted that many other methods exist to estimate the zero-coupon yield curve such as polynomial, discount function and cubic spline methods. Thus, the estimates as well as the results may vary depending on the type of method that is chosen. The scope of this paper does not include the revision of the different methods and their accuracy. Therefore, since I used the estimated zero-coupon Yield Curve derived from the Nelson-Siegel-Svensson method, my results may be conditioned on such method's peculiarities .

- The implied forward rates forecast future spot rates up to 29 years. However, the data provided by the Survey of Professional Forecasters that I used to perform my multivariate linear regression model only provides a one-year forecast. Therefore, only a limited analysis could be performed. Probably, if I forecasts for longer period were available on the macro-indicators than I used (CPI, Nominal GDP and Federal Funds), much more conclusive results could have been obtained.

5. CONCLUSION

The objective of this paper was to determine if the implied forward rates derived from the bond market are a good predictor of the future risk-free rate.

In order to do so, first I outlined the fundamental principles of finance since they are necessary for understanding how finance works. Effects of time and risk in money's value were then derived from the fundamental principles. On the one hand, the value of money is not constant over time. Due to the fact that individuals prefer, *ceteris paribus*, to have money now rather than later, the value of money diminishes the longer it takes to be received. Hence, a dollar today is worth more than a dollar tomorrow.

On the other hand, individuals are risk averse. Therefore, they demand higher returns to riskier investments. Thus, the value of any asset or investment is largely determined by the time and degree of risk that is involved.

Normally, valuation methods isolate the effects of time on money's value by using the rate of return of a risk-free asset, that is, the risk-free rate. Although the risk-free rate is a theoretical concept, zero-coupon government bonds can be used to derive the risk-free rate. That is because such securities practically have no default risk nor reinvestment risk.

Once the relationships between the yield curve, spot rates, forward rates and future rates were explained I decided to perform a quantitative analysis using historical data provided by the U.S. Board of Governors of the Federal Reserve System. First, I calculated the implied forward rates of zero-coupon bonds ranging from 1 to 5 maturity years. Then, using historical values, I was able to observe the errors that may arise between the implied forward rates and the actual future rates.

Then I used a linear mixed effects model to obtain statistical results on the historical errors. My model indicated that the length of the forecast and the years to maturity of the bond are significant at all confidence levels when it comes to explaining errors. To be more precise, a positive non-linear relationship exists between the length of the forecast and the implied forward rates' errors. In fact, such a result is reasonable: an estimate one year forward is often more precise than an estimate twenty-years forward.

On the other hand, a negative linear relationship exists between the years to maturity of the bond and the implied forward rates' errors. That is, the implied forward rates' errors tend to be lower for long-term bonds than for short-term bonds. Taking into account that a long-term rate of return is nothing but the sum of weighted averages of short-term rates, the effects of external shocks can be smoothened through the different years of the long-term bond while in a short-term bond this might not be possible.

Eventually, a multivariate linear regression model was performed to determine why the implied forward rates' errors may arise. The results of my model indicated that changes in the federal funds rate have a strong positive correlation with the implied forward rates' errors. Such a result is consistent with the bond market behavior: when the federal funds rate is increased, newly issued government' securities also increase their interest rate to make them more attractive to investors. That leads to an increase in the risk-free rate. However, the results on the rest of the variables were inconclusive.

In conclusion, the implied forward rates' errors depend on the length of the forecast and the type of bond. The implied forward rates of long-term bonds are more accurate than those of short-term bonds. In addition, the shorter the length of the forecast, the more accurate the implied forward rates are. Taking this into account along with the easy access to the data of implied forward rates, implied forward rates can be used as a reliable indicator of the future risk-free rate if time and/or resources are a constraint.

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